Grade 4 Mathematics, Quarter 1, Unit 1.1

Understand, Read, Write, and Use Place Value up to 1,000,000

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Understand the place value of the base ten number system to 1,000,000.
- Represent whole numbers using expanded form/notation.
- Compare whole numbers using symbols.
- Read and write numbers to 1,000,000.
- Round any number to 1,000,000.
- Understand the relationship between two digits in a multi-digit number by applying the concept of place value and division.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Students make sense of quantities using place value and their relationships among the digits in a number in problem situations by decontextualizing and contextualizing a variety of math problems.

Attend to precision.

- Students communicate precisely the value of numbers and digits to others using clear definitions in discussion.
- Students will state the meanings of the symbols they choose as they appropriately and consistently compare numbers.
- Students give carefully formulated explanations as the write, read, compare, and round numbers.

Look for and make use of structure.

- Students look closely to discern a pattern or structure as they study place value.
- Students can step back for an overview and shift perspective when problem solving using place value.

Essential questions

- How do you find the place value of any digit in a number?
- How do you use expanded form/notation to represent any number?
- How do you name numbers using the place value system?
- How do you use symbols to compare whole numbers?
- How can you use the base-ten system to read and write any number to 1,000,000?
- How do you round numbers to any place value?
- What is the relationship between the digits in a whole number?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten[2] 4.NBT

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Generalize place value understanding for multi-digit whole numbers.

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*

4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form.

Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In third grade, students extended their understanding of place value up to 1,000; rounded whole numbers to the nearest 10 or 100; and added and subtracted numbers up to 1,000.

Current Learning

In fourth grade, students extend their understanding of place value in a multi-digit whole number; read and write whole numbers using base ten numerals, number names, and expanded form; compare multi-digit numbers based on greater than, less than, or equal to; and round multi-digit numbers to any place up to 1 million.

Future Learning

In fifth grade, students will understand the place value system in base ten and perform operations of multi-digit whole numbers with decimals to the thousandths using exponents.

Additional Findings

According to *Principles and Standards for School Mathematics* by the National Council of Teachers of Mathematics, "Students who understand the structure of numbers and the relationships among numbers can work with them flexibly." (p. 149)

The methods used for comparing and rounding numbers in previous grades apply to these numbers because of the uniformity of the base 10 system. (*Progressions* grade 4, p. 12, paragraph 3)

Grade 4 Mathematics, Quarter 1, Unit 1.2

Adding and Subtracting Whole Numbers up to 1,000,000

Overview

Number of instructional days: 7 (1 day = 45–60 minutes)

Content to be learned

- Add multi-digit whole numbers using the standard algorithm.
- Subtract multi-digit whole numbers using the standard algorithm.
- Develop understanding of the standard algorithm for addition and subtraction computations.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and relationships in problem situations while adding and subtracting numbers by decontextualizing and contextualizing a variety of math problems.

Look for and make use of structure.

- Look closely to discern a pattern or structure as they develop standard algorithms when adding and subtracting numbers.
- Step back for an overview and shift perspective when problem solving as they add and subtract numbers.

Look for and express regularity in repeated reasoning.

- As they begin to form and understand algorithms for adding and subtracting numbers.

Essential questions

- What is an algorithm?
- How do you add multi-digit whole numbers using the standard algorithm?
- How do you subtract multi-digit whole numbers using the standard algorithm?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten[2] 4.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In third grade, students developed their understanding of numbers by building their facility with mental computation (adding and subtracting within 1,000) and using strategies and algorithms based on place value and properties of operations.

Current Learning

In the fourth grade, students add and subtract multi-digit whole numbers using the standard algorithm.

Future Learning

In fifth grade, students will understand the place value system and use place value to reinforce performing operations with multi-digit whole numbers and with decimals to thousandths. They will make use of the standard algorithm in adding and subtracting multi-digit whole numbers.

Additional Findings

"Conceptual understanding and procedural fluency with multi-digit numbers and decimal fractions require that students understand and use the base 10 quantities represented by number words and number notation." National Research Council *Adding it Up: Helping Children Learn Mathematics.* (p. 417)

"Multi-digit addition and subtraction knowledge seems to consist of different pieces that children put together in different orders and in different ways." (p. 78 e.g., Hiebert & Wearne, 1986) *A Research Companion to Principles and Standards for School Mathematics*

Grade 4 Mathematics, Quarter 1, Unit 1.3

Identify Factors, Multiples, Prime, and Composite Numbers from 1–100

Overview

Number of instructional days: 8 (1 day = 45–60 minutes)

Content to be learned

- Identify a multiplication equation as a comparison.
- Verbally interpret statements of multiplicative comparisons as multiplication equations.
- Find factor pairs for whole numbers ranging from 1–100.
- Recognize that a whole number is a multiple of all its factors.
- Determine whether a given whole number ranging from 1–100 is a multiple of a given one-digit number.
- Determine whether a whole number ranging from 1–100 is prime or composite.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships as they interpret multiplication problem situations.

Model with mathematics.

- Apply the mathematics of multiplication they know to solve problems arising in everyday life, society, and the workplace.
- Analyze number relationships using factors and multiples to draw conclusions.

Look for and make use of structure.

- Discern a pattern or structure as they learn about the relationships in multiplication, and in finding factors and multiples of a number.
- Notice patterns or structures as they learn that 35 is 5 groups of 7 or 7 groups of 5.
- Notice patterns found in prime and composite numbers as they begin to identify these numbers.

Essential questions

- How is a multiplication equation also a comparison?
- How would you write 4 times as many groups of 6 using as equation?
- How do you find the factor pairs for any given whole number between 1–100?
- How would you describe the relationship of factors and their multiples?
- How would you find the multiples of any one-digit number?
- What is a prime number?
- What is a composite number?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.

Represent verbal statements of multiplicative comparisons as multiplication equations.

Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1–100.

Recognize that a whole number is a multiple of each of its factors.

Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number.

Determine whether a given whole number in the range 1–100 is prime or composite.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of

the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Students in second grade began working with equal groups of objects to build a foundation for multiplication. Third-grade students developed an understanding of the meaning of multiplication, first as repeated addition. Students later transitioned to multiplicative thinking to solve problems involving equal-sized groups, arrays, and area models. Students also solved problems in which they identified and explained patterns in arithmetic terms.

Current Learning

Students in fourth grade interpret multiplication equations as comparisons, and are able to make verbal statements about the multiplication equations. They develop an understanding of the relationship between factors and multiples of whole numbers, and are able to generate and identify both factors and multiples of numbers ranging from 1–100. Students apply their understanding of factors and multiples to enhance multiplication and division strategies. Students master the concepts of factors, prime, and composite numbers in grade 4.

Future Learning

Students in fifth grade will develop knowledge of order of operations to analyze increasingly complex equations and relationships in multiplication equations.

Additional Findings

A Research Companion to Principles and Standards for School Mathematics, "Finding and using patterns greatly simplifies the task of learning multiplication combinations. It is one of the very essences of mathematics. Thus, approaching multiplication learning as pattern finding greatly simplifies the task and constitutes a core mathematical approach." (p. 77)

Adding It Up: Helping Children Learn Mathematics, "Using patterns and other thinking strategies greatly simplifies the task of learning multiplication. Finding and describing patterns are a hallmark of mathematics. Thus, treating multiplication learning as pattern finding both simplifies the task and uses a core mathematical idea. (p. 192)

Grade 4 Mathematics, Quarter 1, Unit 1.4

Use Multiplication Strategies to Accurately Solve Problems

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Solve word problems using multiplication.
- Multiply a whole number of up to four digits by a one-digit whole number using properties of operations and place value.
- Model and explain multiplication using equations, rectangular arrays, or area models.
- Solve multistep word problems with whole number answers.
- Represent problems using equations with a letter standing for the unknown quantity.
- Identify the difference between a multiplicative and additive comparison.
- Assess reasonableness of answers through mental computation, estimation, and rounding with multiplicative comparisons.
- Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.

Essential questions

- How would you determine if your answer is reasonable through estimation?
- How do you use rounding to check for reasonableness?
- When solving word problems, how can you tell when to use addition or when to use multiplication?
- How would you multiply a whole number of up to four digits by a one-digit whole number?
- How do you apply the area and perimeter formulas for rectangles in real-world and mathematical problems?

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations as they multiply numbers.
- Use rounding to evaluate the reasonableness of answers.

Construct viable arguments and critique the reasoning of others.

- Construct plausible arguments with the use of appropriate graphic organizers as they explain multiplication and their models used.
- Reason about models used in multiplication situations and justify conclusions.

Model with mathematics.

- Students can apply multiplication to solve problems arising in everyday life, society, and the workplace.
- They can analyze multiplicative relationships to draw conclusions about the situation.
- How can you use equations, rectangular arrays, or area models to multiply?
- How do you solve multistep word problems?
- How can you solve a problem with the letter standing for the unknown quantity?
- What is another way to represent 46 + 46 + 46 + 46?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]

Number and Operations in Base Ten[2] 4.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, ~~and~~ multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.

Represent these problems using equations with a letter standing for the unknown quantity.

Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

[1] See Glossary, Table 2.

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In grade 3, students understood the meanings of multiplication of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for

multiplication). They applied properties of operations as strategies to multiply.

Current Learning

In grade 4, students apply their understanding of models for multiplication to multiply multidigit whole numbers (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication). They apply appropriate methods to accurately solve multiplication problems, including word problems. They move from solving problems using additive reasoning to multiplicative reasoning.

Future Learning

In grade 5, students will appropriately apply their understanding of multiplication to solve division problems. Students will use multiplication and division with decimals to the thousandths.

Additional Findings

According to *Adding It Up: Helping Children Learn Mathematics*, National Research Council, "Problem solving should be the site in which all of the strands of mathematics proficiency converge. It should provide opportunities for students to weave together the strands of proficiency and for teachers to assess students' performance on all of the strands." (p. 421)

The book also states, "An important part of our conception of mathematical proficiency involves the ability to formulate and solve problems coming from daily life or other domains, including mathematics itself." (p. 420)

As stated in *Principles and Standards for School Mathematics*, "Students who understand the structure of numbers and the relationships among numbers can work with them flexibly" (p. 149).

Grade 4 Mathematics, Quarter 2, Unit 2.1

Multiplying Two-Digit Numbers

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Multiply a whole number of up to four digits by a one-digit whole number.
- Multiply two 2-digit numbers.
- Multiply using equations, rectangular arrays, or area models.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Construct plausible arguments with the use of appropriate graphic organizers.
- Analyze situations and justify conclusions.

Model with mathematics.

- Apply mathematics to solve problems arising in everyday life, society, and the workplace.
- Analyze relationships mathematically to draw conclusions.

Look for and make use of structure.

- Look closely to discern a pattern or structure.
- Step back for an overview and shift perspective when problem solving.

Essential questions

- How would you multiply a whole number of up to four digits by a one-digit whole number?
- What is the process for multiplying two 2-digit numbers?
- How would you use equations, rectangular arrays, or area models to multiply?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten[2] 4.NBT

[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Common Core State Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In third grade, students understood the meaning of multiplication of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines). They applied properties of operations as strategies for multiplying.

Current Learning

In fourth grade, students reinforce their understanding of models for multiplication (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication) to multiply multi-digit whole numbers.

Future Learning

In fifth grade, students will fluently multiply multi-digit numbers and appropriately apply strategies to solve division problems. Students will use multiplication and division with decimals to the thousandths.

Additional Findings

According to *Principles and Standards for School Mathematics: National Council of Teachers of Mathematics*, "as students develop methods to solve multi-digit computation problems, they should be encouraged to record and share their methods" (p. 153), and "as students move from third to fifth grade, they should consolidate and practice a small number of computational algorithms for addition, subtraction, multiplication and division that they understand well and can use routinely" (p. 155).

Grade 4 Mathematics, Quarter 2, Unit 2.2

Find Whole-Number Quotients and Remainders with up to 4-Digit Dividends and 1-Digit Divisors

Overview

Number of instructional days: 11 (1 day = 60 minutes)

Content to be learned

- Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors.
- Use division strategies based on place value, properties of operations, and the relationship between multiplication and division.
- Illustrate and explain the calculation using equations, models, and arrays.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Understand and use stated assumptions and definitions and establish results in constructing arguments.
- Justify conclusions, communicate them to others, and respond to the arguments of others.

Model with mathematics.

- Model by illustrating using equations, arrays, and models.

Look for and make use of structure.

- Recognize the relationship between multiplication and division.

Essential questions

- How can you solve division problems with or without remainders?
- How can multiplication help you to divide?
- What are some strategies for solving a division problem with or without remainders.

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten[2] 4.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Common Core State Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may

sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In third grade, students worked on division within 100. They fluently multiplied and divided within 100 using strategies. Students understood the properties of multiplication and the relationship between multiplication and division.

Current Learning

In fourth grade, students find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. Students use strategies based on place value, properties of operations, and relationships between multiplication and division. Students illustrate and explain using equations, arrays, and /or models.

Future Learning

In fifth grade, students will develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. Students will be fluent with multi-digit addition, subtraction, multiplication, and division. They will use previous knowledge to apply their understanding to decimals.

Additional Findings

According to *Adding it up: Helping Children Learn Mathematics*, "The many kinds of errors students make when multi-digit methods are not connected to place-value meanings are well documented." (p. 213)

A Research Companion to Principles and Standards for School Mathematics states that "the most powerful problem-solving approach is to understand the situation deeply—that is, to be able to draw it or otherwise represent it to oneself." (p. 69)

Grade 4 Mathematics, Quarter 2, Unit 2.3

Use Operations With Whole Numbers to Solve Problems

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Multiply or divide to solve word problems.
- Assess the reasonableness of answers using mental computation and estimation.
- Generate and analyze patterns.
- Identify apparent features of patterns.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Demonstrate ability to decontextualize and contextualize involving quantitative relationships.

Model with mathematics.

- Apply mathematics to solve problems in everyday life.
- Identify important quantities in a practical situation.
- Analyze relationships mathematically to draw conclusions.

Look for and make use of structure.

- Look closely to discern a pattern or structure.
- Step back for an overview and shift perspective.
- See complicated things as being composed of several objects.

Essential questions

- What are the steps in solving a multiplication or division word problem?
- How would you solve problems using mental computation and estimation?
- How would you generate and analyze patterns?
- How would you identify apparent features of patterns?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]

[1] See Glossary, Table 2.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions

and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In third grade, students began to understand the relationship between multiplication and division as a part of algebra readiness. They created and analyzed patterns and relationships involving multiplication and division.

Current Learning

In fourth grade, students continue to identify, describe, and extend numeric patterns involving all operations and non-numeric growing or repeating patterns. Through these experiences, they develop an understanding of the use of a rule to describe a sequence of numbers or objects. They solve multistep problems that arise in mathematics and reflect on the processes of mathematical problem solving.

Future Learning

In fifth grade, students will continue to use patterns, models, and relationships as context for writing and solving simple equations and inequalities to graph ordered pairs on a coordinate plane.

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics. National Council of Teachers of Mathematics,* "the goal of problem solving is for students to use problem situations as gateways to abstraction and generalization—to develop the ability to mathematize situations." (p. 263)

According to *Principles and Standards for School Mathematics*, "young students can engage in substantive problem solving and in doing so develop basic skills, higher order thinking skills, and problem-solving strategies" (Cobb et al. 1991; Trafton and Hartman 1997). (p. 121)

Grade 4 Mathematics, Quarter 3, Unit 3.1

Understanding and Comparing Fractions, Equivalence, and Ordering

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Explain why a fraction is equivalent to another fraction using visual fraction models.
- Recognize and generate equivalent fractions.
- Compare two fractions with different numerators and different denominators by creating common denominators or numerators.
- Compare two fractions with different numerators and different denominators by comparing to a benchmark fraction such as 1/2.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Create a coherent representation of the problem at hand; considering the units involved in attending to the meaning of quantities.

Construct viable arguments and critique the reasoning of others.

- Construct arguments using objects, diagrams, drawings, and actions.

Look for and make use of structure.

- Look closely to notice a pattern or structure.

Essential questions

- How can you show that two fractions are equivalent using a visual fraction model?
- What strategies can be used to generate equivalent fractions?
- How can you compare two fractions with like numerators and unlike denominators?
- How can you compare two fractions with like denominators and unlike numerators?
- How does a benchmark fraction, such as 1/2, help you compare fractions?

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3] **4.NF**

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.

Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2.

Recognize that comparisons are valid only when the two fractions refer to the same whole.

Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or

reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are notgeneralized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In third grade, students developed an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points and distances on a number line. They understood and used models, including the number line, to identify equivalent fractions. They solved problems that involved comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators.

Current Learning

In fourth grade, students are able to extend their understanding of fraction equivalence and ordering. They are building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. Students also compare to fractions with different numerators and denominators by creating common denominators or by comparing to a benchmark fraction. They recognize that these are only valid when two fractions refer to the same whole.

Future Learning

In fifth grade, students will use equivalent fractions as a strategy to add and subtract fractions. They will also apply and extend previous understandings of multiplication and whole number fraction division to multiply and divide fractions by a whole number.

Additional Findings

Principles and Standards for School Mathematics states. “By using an area model in which part of a region is shaded, students can see how fractions are related to a unit whole, compare fractional parts of a whole, and find equivalent fractions.” (p. 150)

The book also states, “By using parallel number lines, each showing a unit fraction and its multiples, students can see fractions as numbers, note their relationship to 1, and see relationships among fractions, including equivalence.” (p. 150)

Grade 4 Mathematics, Quarter 3, Unit 3.2

Addition and Subtraction of Fractions with Like Denominators

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Add and subtract fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Understand addition and subtraction of fractions as joining and separating fraction parts referring to the same whole.
- Decompose a fraction into a sum of fractions with the same denominators in more than one way.
- Justify decomposition by using a visual fraction model.
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.
- Make a line plot to display a data set of measurements in fractions.
- Fluently add and subtract multi-digit whole numbers.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Rely on using concrete objects or pictures to help conceptualize and solve a problem**.**
- Check answers to problems using different methods**.**
- Solve complex problems and identify correspondences between different approaches**.**

Use appropriate tools strategically.

- Consider the available tools when solving a mathematical problem**.**
- Technology can enable them to visualize the results of varying assumptions**.**

Look for and express regularity in repeated reasoning.

- Notice if calculations are repeated**.**
- They maintain oversight of processes, while attending to the details.

Essential questions

- How do add and subtract fractions to make a whole with same denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100?
- What does it mean to join and separate fractional parts referring to the same whole?
- How do you decompose a fraction into parts with the same denominator to make a whole?
- In what ways can you solve word problems involving adding and subtracting fractions with like denominators?
- How do you display a fractional set of data on a line plot?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3] 4.NF

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Measurement and Data 4.MD

Represent and interpret data.

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8).

Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In third grade, students solved problems that involved comparing and ordering fractions with denominators 2, 3, 4, 6, and 8 by using models, benchmark fractions, or common numerators or denominators (e.g., 3/8 = 1/8 + 1/8 +1/8). They understood and used models, including the number line, to identify equivalent fractions.

Current Learning

In fourth grade, students build fractions by applying and extending previous understandings of operations (understand adding and subtracting fractions with mixed numbers and with like denominators 2, 3, 4, 6, 8, 10, 12, and 100) on whole numbers. Students decompose fractions in sum of fractions and solve word problems involving addition and subtraction of fractions. Students in fourth grade display a data of measurements in fractions of a unit on line plots. Students will continue to add and subtract multi-digit whole numbers to build fluency. Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Future Learning

In fifth grade, students will apply their understandings of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with unlike denominators. They will develop fluency with standard procedures for adding and subtracting fractions and decimals. Students will add and subtract fractions and decimals to solve problems, including problems involving measurement.

Additional Findings

Principles and Standards for School Mathematics states, "Through the study of various meanings and models of fractions—how fractions are related to each other and to the unit whole and how they are represented—students can gain facility in comparing fractions, often by using benchmarks such as 1/2 or 1" (p. 149).

According to *Adding It Up*, "Reading between the data requires students to compare quantities and use mathematical operations to combine and integrate data and to identify mathematical relationships expressed in the data or in visual representations of the data" (p. 290).

Principles and Standards for School Mathematics also states, "Computational fluency refers to having efficient and accurate methods for computing" (p. 152).

Grade 4 Mathematics, Quarter 3, Unit 3.3

Apply and Extend Understanding of Multiplication to Multiply a Fraction by a Whole Number

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Apply and extend previous understanding of multiplication to multiply a fraction by a whole number.
- Understand the fraction *a/b* as a multiple of *1/b* by using a visual fraction model and recording a conclusion with an equation [e.g., 5/4 = 5 x (1/4)].
- Understand of multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5.
- Solve word problems involving multiplication of a fraction by a whole number.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Rely on using concrete objects or pictures to help conceptualize and solve a problem**.**
- Check answers to problems using different methods**.**
- Solve complex problems and identify correspondences between different approaches**.**

Essential questions

- How can you apply and extend previous understanding of multiplication to multiply a fraction by a whole number?
- How can you explain that a fraction a/*b* is a multiple of 1/*b*?
- How can you explain that a multiple of a/*b* is a multiple of 1/*b*, and how can you use this understanding to multiply a fraction?
- How can you solve word problems involving multiplication of a fraction by a whole number?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3] 4.NF

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a/b* as a multiple of *1/b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

b. Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In third grade, students developed an understanding of fractions as numbers. They explained equivalence of fractions, expressed whole numbers as fractions, and compared fractions by reasoning about their size.

Current Learning

In fourth grade, students are reinforcing their understanding of fractions and fraction equivalence. Students are developing an understanding of the addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

Future Learning

In fifth grade, students will develop fluency with addition and subtraction of fractions, and develop understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by fractions).

Additional Findings

According to the *PARCC Progressions 3–5 Number and Operations/Fractions*, "in general, they (the students) see a fraction as the numerator times the unit fraction with the same denominator. And, "The same thinking, based on the analogy between fractions and whole numbers, allows students to give meaning to the product of a whole number and a fraction" (p. 8).

According to *A Research Companion* to *Principles and Standards for School Mathematics*, "Students require a constant intertwining of understanding and doing – of building meaning, of problem solving, and of computing" (p. 91).

Grade 4 Mathematics, Quarter 3, Unit 3.4

Compare and Understand Decimals to the Hundredths Place

Overview

Number of instructional days: 10 (1 day = 60 minutes)

Content to be learned

- Express a fraction with denominator 10 as an equivalent fraction with denominator 100.
- Understand decimal notation for fractions and compare decimal notation.
- Use decimal notation for fractions with denominators 10 or 100 to rewrite 0.62 as 62/100.
- Compare two decimals to hundredths by reasoning about their size.
- Justify conclusions by using a visual model.

Mathematical practices to be integrated

Reason abstractly and quantitatively**.**

- Make sense of quantities and their relationships in problem situations**.**
- The ability to decontextualize; to represent a given situation symbolically**.**
- The ability to contextualize; to pause as needed**.**

Use appropriate tools strategically.

- They consider the available tools when solving a mathematical problem with pencil and paper, concrete models, a ruler, a calculator, or spreadsheet.
- Use technological tools to explore and deepen their understanding of concepts.

Look for and make use of structure.

- Recognize patterns and structure in the place value system.
- Recognize patterns and structure in adding and subtracting multidigit whole numbers.

Essential questions

- How do you express a fraction with denominator 10 as an equivalent fraction with denominator 100?
- How do you use decimal notations for fractions to compare decimal notations?
- How can you compare two decimals to the hundredths by reasoning about their size?
- How can you use a visual model to justify conclusions?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions[3] 4.NF

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

[4] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

4.NF.7 Compare two decimals to hundredths by reasoning about their size.

Recognize that comparisons are valid only when the two decimals refer to the same whole.

Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In third grade, students developed an understanding of fractions as numbers. They explained equivalence of fractions, expressed whole numbers as fractions, and compared fractions by reasoning about their size.

Current Learning

In fourth grade, students develop understanding of decimal notation for fractions, and compare decimal fractions using visual models to justify a conclusion.

Future Learning

In fifth grade, students will apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They will develop fluency in these computations, and make reasonable estimates of their results. Students will use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers, to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

Additional Findings

According to the PARCC Progressions, fractions with denominators 10 and 100, called decimal fractions, arise naturally when students convert between dollars and cents. For example, because there are 10 dimes in a dollar, three dimes is 3/10 of a dollar; and it is also 3/100 of a dollar because it is 30 cents, and there are 100 cents in a dollar" (p. 8).

According to the PARCC Progressions, students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator" (p. 9).

Grade 4 Mathematics, Quarter 4, Unit 4.1

Draw and Identify Lines

Overview

Number of instructional days: 13 (1 day = 45–60 minutes)

Content to be learned

- Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines.
- Identify points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines in two-dimensional figures.
- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
- Recognize and identify line-symmetric figures.
- Draw lines of symmetry.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions.

Use appropriate tools strategically.

- Students use tools that might include a ruler, a protractor, or dynamic geometry software to solve a mathematical problem.

Attend to precision.

- Students try to communicate precisely to others and give carefully formulated explanations to each other.

Essential questions

- How would you describe points?
- What is the difference between a line and a line segment?
- How are rays and angles (right, acute, and obtuse) related?
- What is the difference between perpendicular and parallel lines?
- Given a two-dimensional figure, how do you identify perpendicular and parallel lines?
- How do you identify a line symmetry?
- How do you classify shapes by properties of their lines and angles?
- How can you recognize and identify line-symmetric figures and draw lines of symmetry?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and

compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In third grade, students compared and classified shapes by their shape and angles, which are their attributes, and connected these with their names and definitions of shapes.

Current Learning

In fourth grade, students understand that two-dimensional geometric figures can be classified based on properties such as particular angle measures (right, obtuse, acute). Students recognize angles as geometric shapes that are formed wherever two rays share a common endpoint. Students also classified shapes by perpendicular and parallel lines. Students recognize and classify line symmetry into matching parts.

Future Learning

In fifth grade, students will classify and categorize shapes based on attributes such as angle measure and use a pair of perpendicular lines to define a coordinate system.

Additional Findings

Principles and Standards for School Mathematics states, "As students sort, build, draw, model, trace, measure, and construct, their capacity to visualize geometric relationships will develop." (p. 165)

The book also states, "The reasoning skills that develop in grades 3–5 allow them to investigate geometric problems of increasing complexity and to study geometric properties." (p. 165)

Grade 4 Mathematics, Quarter 4, Unit 4.2

Draw, Identify, and Measure Angles

Overview

Number of instructional days: 14 (1 day = 45–60 minutes)

Content to be learned

- Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand the concepts of angle measurement.
- Recognize the angle measure as the sum of one-degree angles.
- Measure and sketch angles in whole-number degrees using a protractor.
- Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically—and the ability to manipulate the representing symbols as if they have a life of their own.

Attend to precision.

- Students are careful about specifying units of measure.

Look for and make use of structure.

- Recognize the significance of an existing line in a geometric figure and use the strategy of drawing an auxiliary line for solving problems.

Essential questions

- How can you recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand the concepts of angle measurement?
- How can you recognize the angle measure as the sum of one-degree angles?
- How can you measure and sketch angles in whole-number degrees using a protractor?
- How can you solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data 4.MD

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated

explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In second grade, students recognized and drew shapes having specified attributes such as a given number of angles. In third grade, students compared and classified shapes by their shapes and angles, and connected these with definitions of shapes.

Current Learning

In fourth grade, students understand that geometric figures can be classified based on properties such as particular angle measures. Students recognize angles as geometric shapes that are formed wherever two rays share a common endpoint.

Future Learning

Fifth graders will not address angles, but they will recognize volume as an attribute of three-dimensional shapes as part of geometric measurement.

Additional Findings

Principles and Standards for School Mathematics states, "Students learn that a square corner is called a right angle and establish this as a benchmark for estimating the size of other angles" (p. 172)

The book also states, "Students in grades 3–5 should encounter the notion that measurements in the real world are approximate." (p. 172)

Grade 4 Mathematics, Quarter 4, Unit 4.3

Solve Problems Involving Measurement and Conversion of Measurements from a Larger Unit to a Smaller Unit

Overview

Number of instructional days: 14 (1 day = 45–60 minutes)

Content to be learned

- Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec.
- Record measurement equivalents in a two-column table.
- Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money involving problems including fractions and/or decimals.
- Represent measurement quantities using diagrams such as number lines that feature a measurement scale.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships; graph data, and search for regularity or trends.

Use appropriate tools strategically.

- Consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, or a calculator.

Attend to precision.

- Students are careful about specifying units of measure and try to communicate precisely those units of measurement to others.

Essential questions

- How do you identify and compare measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec (*e.g., recognize that 1 foot is 12 times as long as 1 inch)?*
- How do you create a two-column table to compare and record measurement equivalents?
- How do you use addition, subtraction, multiplication, and division to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money problems including fractions and/or decimals?
- How do you demonstrate measurement quantities using diagrams such as number lines that include a measurement scale?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data **4.MD**

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In third grade, students measured time intervals in minutes; solved word problems involving addition and subtraction; and represented these on a number line diagram. Students also measured and estimated liquid volumes and masses of objects using standard units of grams, kilograms, and liters. They added, subtracted, multiplied and divided to solve one-step word problems involving masses or volumes given in the same units by using drawings to represent the problem.

Current Learning

In fourth grade, students know relative sizes of measurement units within one system of units (km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec). They express measurement in a larger unit in terms of a smaller unit. They record measurement equivalents in a two-column table. Students use the four operations to solve word problems involving distances, intervals of time, liquid volumes, and masses of objects, and money, including problems involving simple fractions or decimals. Students represent measurement quantities using diagrams such as a number line. Students also apply the area and perimeter formulas for rectangles in real-world mathematical problems.

Future Learning

In fifth grade, students will convert among different-sized standard measurement units within a given measurement system and use these conversions in solving multi-step, real-world problems. Students will

represent and interpret data. In geometric measurement, they will understand concepts of volume and relate volume to multiplication and division.

Additional Findings

According to *Principles and Standards for School Mathematics*, "Learning how to choose the appropriate unit is a major part of understanding measurement." (p. 45)

The book also states, "Students should learn both customary and metric systems." (p. 45)